

The Third Round

Written by bobthefam
and the CALT committee

Thanks to

djmathman, kootrapali, IMadeYouReadThis, lrjr24, and nikenissan

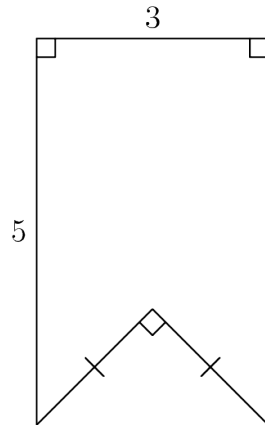
for testsolving

Rules

1. You have ten minutes to complete this test.
 2. You are permitted the use of scratch paper, rulers, protractors, and compasses. No online resources or calculators are allowed.
 3. All answers will be integers.
 4. Discussion of this test is not permitted.
 5. Figures are not necessarily drawn to scale.
 6. SCORING: Each question is worth 1 point.
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1. A number is called *open* if its one's digit is a perfect square that is not 0. A number is called *cubedd* if it is divisible by a perfect cube that is not 1. What is the largest 4-digit number that is both *open* and a *cubedd*?
2. If 3 dots are equal to 5 mots, how many mots equal 7 dots? If the answer is expressed as $\frac{a}{b}$, where a and b are relatively prime, find $a + b$.
3. A square is placed on the coordinate plane. Two opposite vertices of the square are $(27, 7)$ and $(19, 31)$. What is the area of the square?
4. What is the square root of the sum of the coefficients of the expansion of $(a + 2b + 3c + 4d)^6$?
5. If John has 12 coins, each with a 50% chance of landing on tails or heads, what is the probability that he gets more heads than tails? If the value can be expressed as $\frac{a}{b}$, what is the value of $b - a$?
6. Joe has a 30 question test. He gets 5 points for each correct answer and -2 points for each wrong answer. He gets a score of 24. If x is the number of correct questions, and y is the number of incorrect questions, what is the value of $xy + x + y + 1$?
7. The area of the banner shown below can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a - b$.



8. What is the largest integer n such that $2^{869} > 10^n$?
9. An ellipsoid has 3 axes. Another ellipsoid has an axis that is half the axis of the first ellipsoid's first axis, and has a second axis that is $\frac{2}{5}$ the axis of the first ellipsoid's second axis. If the second ellipsoid has ten times the volume of the first ellipsoid, what is the ratio of the second ellipsoid's third axis to the first ellipsoid's third axis? The volume of an ellipsoid is $\frac{1}{6}abc\pi$, where a , b , and c are the lengths of the ellipsoid's axes.
10. The area of bobthefam in normal 12-pt font is 0.75 cm. If I want to make a mini 3-d sculpture of my username in 96-pt font, (everything is enlarged by a factor of 8) such that it has a depth of 2 cm, what is the volume of the sculpture? Express your answer in cubic mm.
11. If the graph $y = 3x^2 - 19x + k$, has an x -intercept of 4, what is the value of the sum of the other x -intercept and the value of k ? If the answer is expressed as $\frac{a}{b}$, where a and b are relatively prime, find $a + b$.

12. For which value of n is it true that

$$\frac{1 + 2 + 3 + 4 + \dots + 4n}{3n} = 30?$$

13. An integer y is selected from the interval $[1, 60]$. The probability that y is relatively prime to 60 can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

14. Mel calculates the value of $19 + 12(7)$. As he is not familiar with PEMDAS, he first adds the two numbers and then multiplies by 7. What is the absolute difference between his answer and the correct answer?

15. For which value of n is it true that the convex polygon with n sides also has n diagonals?"

16. How many proper subsets without vowels does the set $\{M, A, T_1, H, C, O, U, N, T_2, S\}$ have?

17. Let $a_1, a_2, a_3, \dots, a_{99}$ be a sequence such that

$$10^{a_i} = \frac{i}{i+1} \quad \text{for every integer } 1 \leq i \leq 99.$$

Find $a_1 + a_2 + a_3 + a_4 + \dots + a_{99}$.

18. The volume of the smallest cube a regular tetrahedron with side length 2020 can be placed completely into can be expressed in simplest radical form as $a\sqrt{b}$. Find the sum of the digits of $a + b$.

19. If $f(x) = \lfloor \sqrt{x} \rfloor$, and $g(x)$ returns the number of perfect squares less than x , what is $f(g(f(2020)))$?

20. What is the ratio of the area of a regular hexagon with side length 6 to an equilateral triangle with side length 8? If the answer is expressed as $\frac{a}{b}$, where a and b are relatively prime, find $a + b$.

21. a and b are relatively prime positive integers such that each of them has at least 3 divisors. What is the smallest possible value of $a + b$?

22. Find the number of digits in the expansion of 2020^{2020} , given that $\log 101 = 2.00432$ rounded.

23. When an unfair coin is flipped 3 times, the expected number of heads is $\frac{15}{7}$. When the coin is flipped twice, what is the expected number of tails? If the answer is expressed as $\frac{a}{b}$, where a and b are relatively prime, find $a + b$.

24. If $15! = 130767_3680_0$, what is the product of the missing digits?

25. Find $x - 2$ if

$$\frac{2020^{2020} - 2020^{2019} - 2020^{2018}}{2020^2 - 2021} = 2020^x.$$

26. What is the sum of the number of faces of all of the platonic solids?

27. Bob picks two integers from 1 to 100, inclusive, with replacement. Let the probability that the product of his two numbers is odd be $\frac{a}{b}$, a common fraction. John does the same thing, but on the second pick, it cannot be the same number as before. Let the probability that the product of his numbers is odd be $\frac{c}{d}$, a common fraction. Find $a + b + c + d$.

28. For all positive integer n , let $f(n)$ denote the closest prime to n . (If two such integers exist, take the larger of the two.) What is the smallest integer n such that

$$f(n+1)^2 - f(n)^2 = 120?$$

29. The Trick'cord Treat Bot, made by Discord Developers, drops an item into the chat every 3.5 minutes. Every message reduces the time left by 2 times the number of letters in the word. If Blob spams a message once every second and his average typing speed is 3 letters per second, how many seconds will he take the Trick'cord Treat Bot to drop an item? Note that the Bot dropped an item and as soon as it was over, Blob started spamming.
30. In the figure shown, the radius of each arc is 2 and all angles that look like right angles are right angles? The area of the shaded region can be expressed in the form $a\pi + b$, where a and b are integers. Find $a + b$.

