CALT April Fool's Contest

Rules

1. You have 6 hours to complete this test. It is due at 4:30 PM PST.

2. You are permitted the use of scratch paper, rulers, protractors, and compasses. No online resources or calculators are allowed.

3. All problems will require a full solution in order to earn points.

4. Each problem is graded out of 7 points.

5. Submit solutions at https://bit.ly/calt-april-fools-contest.

6. If you do not have a discord account, you are not eligible for prizes but you can still get your submission graded by putting "No discord account" in the question. Make sure to join the CALT Discord Server (click on the logo) to stay updated and so you can claim your prize!

February 2021

1. Prove that $(2n - 1)! \equiv n \pmod{n^2}$ if n is prime.

2. Prove that the summation

$$\sum_{a_n=1}^{k} \sum_{a_{n-1}=1}^{a_n} \cdots \sum_{a_2=1}^{a_3} \sum_{a_1=1}^{a_2} a_1$$

   is equivalent to

$$\frac{k(k+1)(k+2)\ldots(k+n)}{(n+1)!}.$$

3. For some positive integers x, 2^x and 5^x have the same leftmost digit, y in the base 10 number system. Find, with proof, the possible values of y.

4. A polyomino is a planar figure formed by joining one or more unit squares edge to edge. How many ways are there to tile a 3 by 3 grid of squares with 6 polyominoes? Rotations and reflections are considered distinct.